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No. XVI.

An Account of Pyrometric Experiments, made at Newark, New Jersey, in April, 1817. By F. R. Hassler.—Read, June 29th, 1817.

THE object of these experiments was to determine the expansion of four iron bars, each two metres in length, and the difference between their expansion and that of brass.

These iron bars are intended to be put end to end, and clamped together in this situation, so as to form one continued bar of eight metres long, fitted in a wooden box, to serve, with certain other apparatus, for the measurement of the base lines in a survey of the coast of the United States, ordered by the government.

The determination of their exact expansion is necessary, in order to reduce the different temperatures observed in the measurement of the bases, to one temperature adopted as a standard; and, in order to compare the length of these bars to the English standard, it is likewise necessary to determine the difference between their expansion and that of brass; the English standard, belonging to the collection of instruments made for the survey of the coast, being a brass scale, of 85 inches long, 2 1-2 inches broad, and half an inch thick, inlaid with silver to receive the divisions, which are tenths of inches over the whole length. It is one of the finest and most accurate pieces of workmanship of the celebrated artist, Mr. Edward Troughton, of London.*

The four iron bars used in the following experiments made together a length of 315,04 English inches, at the temperature of about 50° Fahrenheit; they are 1,1 inch broad, and 0,38 of an inch thick.

To obtain the comparison of their expansion with that of brass, under exactly equal circumstances, I procured the thickest brass wire I could obtain, which was 0,37 of an inch in diameter, and had a length of it straitened, as long as the four bars together. No piece being long enough to make the whole required length, three pieces were jointed and pinned together, overlapping about 2 1-2 inches, as shown in Fig. I., and soldered over the joint, so as to form one single piece of the required length.

The expansion of this length of metal, from the freezing to the boiling point of the thermometer, is out of the reach of any microscopic arrangement; and large enough to allow us to substitute immediate observation for the multiplying apparatus often made use of; since it would give above 1-3 of an inch in the iron, and above 1-2 an inch in the brass, which, as far as I know, is the greatest quantity of expansion as yet submitted to accurate experiment.

To obtain the extent of temperature from freezing to boiling, I chose a season when almost every night brought the temperature of the air near to freezing, so that to obtain the boiling point was the only requisite to be fulfilled by the intended pyrometric arrangement.

I had seen Mr. Troughton, in his pyrometric experiments, use the spirit level upon a lever, resting with one end on its axis, and adjusted at the other by a micrometer screw, so as to measure the increased expansion. Having several spirit

^{*} It may be observed here, that the French standards were always a certain unit of length in iron, and the English standards always a brass scale of inches, on which a mean result is taken, for any length desired. To say more of this belongs to the account of the comparison I have made of these two standards.

212 AN ACCOUNT OF PYROMETRIC EXPERIMENTS,

levels of eight inches long, ground and adjusted by him, and having also the late Mr. Bird's own leveltryer, with a steel micrometer screw, (which Mr. Troughton had made me a present of,) I availed myself of these means to construct a pyrometer, with the accuracy of which I could be satisfied; making use of a brass beam compass with a brass screw, to construct a similar instrument, by adding a crossbar at one end of it, and otherwise suiting it to my purpose. The first, with the steel screw, was used for the iron bars, and the second, with the brass screw, for the brass wire.

The head plate of the screw of Bird's leveltryer is divided into 240 parts, each of which indicates an angular movement of one second of a degree, in the arm of the instrument, which is about seventeen inches long.

I divided a similar plate into the same number of parts, for the brass screw of the new leveltryer.

To determine accurately the absolute value of the revolutions of both screws of these leveltryers, I made one of the pieces cut off from the bars, of near half an inch long, perfectly parallel in two opposite planes, by the same means as the bars themselves had been standarded, and measured its length under the microscope of the brass standard scale above mentioned. This piece was found to be = 0,504543 of an inch. Placing the leveltryer upon an iron plate perfectly even, and adjusting the levels, both on the plate alone, and when this piece was laid under the micrometer screw, I found its value in revolutions and seconds, (which the subdivisions are intended to represent, and which I shall call them hereafter.) Under the steel screw of Bird's level-

tryer, - - = 26R+3'', 1=6243'', 1.Under the brass screw of the new level.

tryer, - - - - = 23R+203,1=5723'',1.This gives the following values for one revolution of the two screws:

For the steel screw, 1R=0.01939581For the brass do. 1R=0.02115815 Decimal parts of an inch. From this I constructed Table I., for the reduction of the observed revolutions and seconds into parts of English inches.

To make the pyrometric experiments with this apparatus, the following arrangements were made.

On the outside of the north wall of the house, an iron bracket was driven into the wall, about five feet from the ground. Upon a cast iron plate, laid on this bracket, a box 2 1-2 inches square on the inside, and of the length of the four bars, was placed vertically, reaching along the wall up to the third story, and fastened to brackets in the wall in various places, without allowing it to touch the wall.

On the ground, and about two feet from one side of the box, a large pot was walled up, in a close oven, to serve as a boiler. The wooden cover of this pot, shutting close, had in its middle a wooden canal or chimney, by which the steam was led into the box, as seen in the lower part of Fig. II. The draught or chimney of the oven was on the opposite side.

In the bottom of the box, on the cast iron plate, was laid a flat piece of ground iron, to give the same level and smooth resting place, both to the bars and the brass wire.

The iron bars and the brass wire were set perpendicularly upon this plate, and held in that position by brass wires, b, b, &c. Fig. II. and III., driven horizontally in the side of the box opposite the cover, about four in each bar's length, and so long that they reached to the cover in b', b', &c. Other smaller brass wires were laid across these, in the following order: nearest to the side of the box was one small cross wire, then came the brass wire under experiment, then again a small cross wire, then the bars, then the brass pins lightly bound together by thin copper wire.

This arrangement, as seen in Fig. VI., formed five intervals between each part, admitted free passage to the steam all round the bars and the wire, hindered the latter from bending by its own weight, prevented their rubbing against the box, or one against the other, and held both perpendicular in their place, and yet so loosely, that when lifted between the wires, they would return again to their exact place; so that their expansion and contraction were perfectly free.

To prevent the iron bars from sliding at their joining places, and at the same time to preserve their ends from rust, which must be the consequence of their exposure to the steam, sheet tin boxes, c, c, of about five inches long, were made to fit the bars exactly, were oiled a little inside, and, when the bars were set upon one another, were slided over the joints.

Fig. II. and III. represent a section of the lower and upper part of the apparatus, through the breadth of the bars and the wire, the whole of it being too long to be represented on a proper scale, and uniform through the whole length, within the size of a common sheet of paper. Fig. V. is a horizontal section of the top, on which the measuring screws rest; and an intermediate section, through a set of supporting pins, is represented in Fig. VI.

Four thermometers were put in the box; one at the top, showing the boiling point just at the upper end of the bars, one about two feet from the bottom, and two at about equal intermediate distances, showing themselves through glasses fitted like windows, in the side f, f, as in Fig. VI.

The temperature being raised in the box by means of the steam from the boiler, which was driven up through the whole extent of the box, the upper ends of the bars and the wire were naturally raised by their expansion, and sunk again by their cooling down to the temperature of the atmosphere.

To measure this expansion by means of the levels above mentioned, they were placed as shown at the top of Fig. III.; Fig. IV. presents a horizontal view of them; t, t, are two iron brackets driven fast into the wall, to receive the resting pins or screws, d, d, d, d, at the ends of the cross bars of the leveltryers; e, e, e, e, are the supporting Y's of the levels L, L; s, s, the screws measuring the expansion on the indices, i, i, by a horizontal stroke, to count them, being made at every two revolutions, reading from the highest division downwards; the subdivisions were read on the divided top plates, g, g, by their coincidence with a vertical line on these indices. The steel micrometer screw rested on the top of the iron bars immediately; but as the wire would not afford a secure rest for the brass screw of its level, and as its divided plate would also have interfered with that of the other screw, a clamp, h, was screwed on the wire, presenting a short brass plate at the distance of about one inch at the side of the wire, to rest this screw upon; the upper plane of this plate being at the height of the bars, so as to give nearly an equal length to the iron and the brass engaged in the experiment. See Fig. III. and V.

To admit a free passage to the steam, so as not to condense it in the box, the top was left open, except a covering of thin muslin round the screws, to prevent the immediate contact of the metal under experiment with the external air.

A screen was placed at x, x, to hinder the steam from reaching the levels, and to make it ascend vertically from the box.

To make the observations thus outside of the house, at the third story, I hung a scaffolding out from the two windows on each side of the apparatus, held at some distance from the wall by butting pieces so as never to touch any part of the box or apparatus, by which means I could walk easily all round it.

First Experiment.

The second of April, the arrangement being ready, fire was made under the boiler, and the water made to boil as much as practicable; but the arrangement being new, and the day cold, the thermometers could not be brought to the boiling point. All that could be obtained, was to bring them to be for some time steady at the following temperatures, reading them in order, from below, upwards.

> Thermometers. 1 2 3 4 Mean. 180°. – 181°. 179°. 5 180°,0.

216 AN ACCOUNT OF PYROMETRIC EXPERIMENTS,

The second thermometer could not be read for want of a fourth assistant.

The levels being adjusted in this temperature, by means of the measuring screws, the indications of the micrometer heads were read as follows :

On the Iron.	On the Brass.
26R + 58''.	24R + 215''.

In this situation the whole apparatus was left, the boiling ceased, and the muslin on the top, and the steam chimney below, removed to let the whole cool.

The 3d of April, at half past 5 o'clock in the morning, the standing of the micrometer heads being verified, and every thing found in order, the levels were again adjusted, and the following readings made on the thermometers and the micrometer heads.

Thermometers.				
1	2	3	4	Mean.
39,5	36,0	36,2	38,3	\$ 37,5.

Micrometers.		
On the Iron.	On the	Brass.
42 r + 20.	47R +	106".

At half past 6 o'clock, returned to the apparatus, adjusted the levels again, and made the following readings.

	The	ermome	eters.	
1	2	3	4	ץ Mean.
39,2		$36,\!5$	36,3	5 37,33.
	M	icrome	ters.	
On the In	on.		Or	n the Brass.
42 R + 28	8''2.		47	'R + 114''.

The day being cold, I could not expect to raise the heat in the box to the temperature of boiling water. I concluded, therefore, to make only an intermediate observation, at the time I should find the thermometers the most equal.

About 7 o'clock in the evening, I made the following observations.

Thermometers.1234Mean. $52^{\circ},0$ $52^{\circ},0$ $51^{\circ},7$ $53^{\circ},0$ 52,2.Micrometers.On the Iron.On the Brass.41R + 42'',5.45R + 91''.

Though I had at the outset doubts upon the admissibility of making observations in the intermediate steps of the rising and falling of the temperature, I still tried it; but found that no coincidence of the thermometers above and below could be obtained, near enough to allow a mean to be taken, except when the temperature in the box was either at the highest degree I could bring it to, or cooled down to the temperature of the surrounding air. To procure this equality of temperature is the principal difficulty in this kind of experiments: For the sensibility of the levels to the expansion and contraction of the iron and brass, is so great, that a change of temperature was often observed in them, before it was observable on the thermometers.

A high wind rising with the night, I was obliged to take in the levels for fear of their being blown down and broken.

Second Experiment.

The 4th of April, at half past 5 in the morning, the temperature of the air being about 32°, I set the levels again in their places, adjusted them, and made the following observations.

> Thermometers. 1 2 3 4 Mean. 36°,0 33°,5 32°,7 32°, 5 33,5. E e

Micron	neters.
On the Iron.	On the Brass.
42 r + 98",5.	48r + 125,"0.

The sun shining on the apparatus, and the day being fair, circumstances seemed favourable for obtaining the temperature of the boiling point. About 8 o'clock, fire was made under the boiler, and the windows for the thermometers shaded from the sun; and, about half past 10, the following heights of thermometers and corresponding readings of the micrometers were obtained, at a constant temperature.

> Thermometers. 1 2 3 4 Mean 212°,5 211°, 211°, 208°, 5 210°,6. Micromotors

Micrometers.					
On the Iron.	On the Brass.				
22 R + 83".	20r + 179".				

The temperature now falling, the fire was raised a second time, and a second reading was made, under circumstances which I found equally trust-worthy, as follows.

	The	rmome	ters.	
1	2	3	4	? Mean.
212°	211°,5	212°	209°	Ś 211°,1.
	Mi	cromet	ers.	

On the Iron.	On the Brass.
2 2r + 25".	20r + 111''.

The boiling being now discontinued, the muslin removed from the top, and the apparatus otherwise left perfectly quiet to cool down to the temperature of the air, at about 8 o'clock in the evening, I adjusted the levels, and observed as follows.

> Thermometers. 1 2 3 4 Mean. 45° 42°,5 44°,8 44°3 44°,15.

Micrometers. On the Brass. On the Iron. 46R + 203''. 41 R + 106'', 5.

The apparatus was now again left in the same position, until next morning.

April 5th, at half past 5 o'clock, every thing being found in good order, I adjusted the levels, and made the following readings.

	Th	ermom	eters.	
1	2	3	4	7 Mean.
35°	32 °	32°	32° .	Š 32°,75.
	N	licrome	ters.	
On th	e Iron.		0	n the Brass.
42 R -	⊢ 118″ .		48	$B_{\rm R} + 112''$.

Then the leveltryer on the iron was removed to clean the screw from rust, and oil it.

Third Experiment.

At 6 o'clock, the arrangement was again mounted, and the levels adjusted, to begin the third set of experiments; and the following readings were made.

Thermometers.				
1	2	3	4	.Mean ز
34°,5	33°,0	33°,0	32°,5	∫ 33°,25.

Micrometers.			
On the Iron.	On the Brass.		
42 R + 170, "5.	48r + 152".		

The boiler being heated, the temperature was raised to the highest about 10 o'clock, and was steady, so that the following readings were made.

	Tł	nermom	eters.	
1	2	3	4	7 Mean.
213°	213°	213°	209°	Š 212°.
		9		

Micrometers.On the Iron.On the Brass.21r + 238''.20r + 62'', 5

'The apparatus was again left quiet, to cool down. At 7 o'clock in the evening, the following readings were made, after adjusting the levels.

	The	ermome	eters.	
1	2	3	4	? M ean.
47°		45°	44°	∫ 45°,3.

Micrometers.

On the Iron.	On the Brass.
40r + 191".	46r + 77",5.

To bring the various observations of these three experiments under a comprehensive view, I have collected them together in Table II., at the end.

The first thing now to be determined, is the length of metal so engaged in these experiments as to influence the standing of the levels.

Besides the length of the bars or the brass wire, an addition is to be made for the length of the micrometer screws of each level, engaged below the bar of the leveltryer.

Then it will be necessary also to add the thickness of the supporting pieces, as they partook, in every case, of the changes of temperature, like the bars. Though these were iron, I think, on account of their smallness, they may, without any alteration, be added to the length of the brass and the iron equally. By these additions, the whole will count from the resting place in the wall, to the bar carrying the level, which also rested on the wall.

The lengths to be used in calculating the results, from the foregoing observations, will be as follows :

For the Iron	l .	For the Brass	J.
Four Bars Screw engaged Plate and Support	Inches. = $315,04$ = $1,1$ = $1,4$	Brass wire Screw engaged Plate and Support	Inches. = $315,1$ = $1,0$ = $1,4$
Sum,	$=\overline{317,54}$	Sum,	= 317,5

To calculate the resulting proportional expansion, for one degree of Fahrenheit's scale, from the foregoing observations, they must be combined together in each experiment, by taking the resulting expansion from the highest degree of heat to any one of the lower degrees.

The quantity of expansion observed, resulting from this comparison of the readings of the micrometer screws, and the reduction of their value by Table I., must be divided by the number of degrees of variation of the temperature, and this must again be divided by the length of the metal under experiment, expressed in the same unit of length as the expansion. The result will be the proportional expansion of the metal, for one degree of Fahrenheit, in a decimal fraction, which may be applied by simple multiplication to any length, and any degree of change of temperature.

This calculation is made by the following extremely simple formula for logarithms.

Log. $P = \log \cdot E + Comp \cdot \log \cdot D + Comp \cdot \log \cdot L$. where P = proportional expansion. E = expansion actually observed. D = degrees of Fahrenheit's scale. L = length of metal under experiment.

So that, for instance, the example of the calculation of the first observation, and result of Table III., at the end, will stand thus:

For	the Ir on.	
$\mathbf{Log.} \ \mathbf{E} = \mathbf{log.}$	0,3072620	= 9,4875088
Com. log. $L = C. \log$.	317,54	=7,4982016
Com. log. $\mathbf{D} = \mathbf{C}$. log.	142,5	= 7,8461851
	0040	1.0010055
Log. $P = \log_{10} 0,0000067$	9040	= 4,8318955

For	the Brass.	
Log. $\mathbf{E} = \log$.	0,4770212	2 = 9.6785441
Com. log. $L = C. \log$.	317,5	=7,4982563
Com. log. $\mathbf{D} = \mathbf{C}$. log.	142,5	= 7,8461851
Log. $P = \log 0.0000108$	5435	= 5,0229855

The details of these combinations of the experiments, and their results, are brought under an easy comprehensive view in Table III., which will be sufficiently explained by the heading of the columns. This table exhibits the agreement of the different results, and the general resulting mean, by taking each observation separately, through the whole of the three experiments. If the mean result of each experiment is taken as one, and the mean of these three results as the final result, the results will be as follows.

First Experiment.

For the Iron. 0,00000679040 679554 713832

Mean 0,00000690808

0,0000105895

For the Brass.

0,0000105435

5443

6808

Second Experiment.

Mean 0,00000698422	0,0000104725
698233	4636
707357	5280
693491	4101
701039	4525
698387	5282
0,00000692024	0,0000104526

222

Third Experiment.

	For the Iron.	For the Brass.
	0,00000707918	0,0000105770
	689014	4187
	Proprietary and a subscription	
Mean	0.00000698466	0.00001049785

Taking a mean of these three results, will present each experiment as one individual result, and give the following general means.

> For Iron = 0,00000695892 For Brass = 0,0000105199 Difference = 0,00000356098

It will be observed, that the two latter experiments give almost an identical mean, from individual results, which also agree very well; that in the first experiment the single results differ more from one another, than in the two latter, and that their mean also differs considerably more than the two others. This first experiment might therefore be rejected if desired; and this I should be inclined to do, because it was the first ever made with the apparatus, when it was new, and its use not familiar, and therefore the results not so trust-worthy. The mean of the two last, would then stand as follows, by taking each experiment for one result.

> For Iron = 0,00000698444 For Brass = 0,0000104851 Difference = 0,00000350066

The single observations of the two last experiments might also be added, to take a general mean, and so a result somewhat different be obtained.

To leave it optional with any person who might wish to make use of these results, to which mean he will give the preference, I will set these different means here together. The difference between them is however only a few units in the 8th decimal, which I think within the limits of accuracy obtainable in this kind of experiments.

Mean Results of Proportional Expansion, for 1° Fahrenheit.

	For Iron.	For Brass.	Difference.
General Mean	= 0,000006963535	0,00001050903	0,000003545495
Mean of three experiments	= 0,00000695892	0,0000105199	0,00000356098
Mean of two last do.	= 0,00000698444	0,0000104851	0,00000350066
Mean of the single observa-			,
tion of the two last expe-	= 0,00000698433	0,0000104789	0,00000349454
riments			

I intended to pursue these experiments further, in the same manner, and to put bar-brass, glass-tubes, iron-wire, &c. under experiment; but the season calling me to the field operations for the survey of the coast, I have postponed them until next winter, when, if circumstances should be favourable, I propose to enter, with greater detail, upon more varied and multiplied results.

F. R. HASSLER.

Newark, New Jersey, June 11, 1817.

TABLE I.

For the Reduction of the Screw Values into Inches.

For the Iron.

For the Brass.

Number.	Revolutions.	Seconds.	Number.	Revolutions.	Seconds.
1	0,01939581	0,000080816	1	0,02115815	0,000088159
2	0,03879162	0,000161632	2	0,04231630	0.000176318
3	0,05818743	0,000242448	3	0,06347445	0.000264477
4	0,07758324	0,000323264	4	0,08463260	0.000352636
5	0,09697905	0,000+0+080	5	0.10579075	0.000410795
6	0,11637496	0,000484896	6	0.12694890	0.000528954
7	0,13577067	0,000565712	7	0.14810705	0.000617113
8	0,15516648	0,000646528	8	0.16926520	0.000705272
9	0,17456229	0,000727344	9	0,19042335	0,000793431

TABLE II.

Experi- ment.		Th	ermome	lers.			Micror	neters.
Number.	1	2	3	4	Mean.	On t	he Iron.	On the Brass.
1st.	180		181	179	180	26 ^R -	- 58″	$24^{R} + 215''$
	39,5	36	36,2	38,3	37,5	42 -	- 20	47 + 106
	39		36,8	36,3	37,3	42 -	- 28,2	47 + 114
	52	52	51,7	53	52,2	41 -	- 42,5	45 + 91
2d.	36	33,5	32,7	32	33,5	42 -	- 98,5	48 + 125
	212	211.5	212	209	211,1	22 -	- 25	20 + 111
	212,5	211	211	208	210,6	22 -	+ 83	20 + 179
	45	42,5	44,8	44,3	44,15	41 -	+ 106,5	46 + 203
	35	32	32	32	32,75	42 -	F 118	48 + 112
3d.	34,5	33	33	32,5	33,25	42 -	+ 170,5	48 + 152
	213	213	213	209	212	21 .	+ 238	20 + 62,5
	47		45	44	45,3	40.	+ 191	46 + 77,5

				I)	I			
Experi- ments.	Ter	mperati	ure.	H	or the Iro	u.	F0	r the Bra	SS.
Number.	from	to	Ex- tent.	Difference of Readings.	Actual Ex- pansion observed.	Proportional Expansion.	Difference of Readings	Actual Ex- pansion observed.	Proportional Expansion.
lst Exp.	180 180	37,5 37,5 57,3	142,5 142,5 197 8	15n + 202' 15 + 210,2 14 - 994 5	0",3072620 0,3079261 0,306845	0,0000679040 679554 713839	22n+131'' 22+139 90+116	0,4777334	0,0000105435 105443 106600
2d Exp.	33,5 33,5 210,6	210,6 211,1 44,15	177,1 177,6 166,4	20 + 15,5 20 + 73,5 19 + 23,5	0,3938561	692024 698387 701039	27 + 187 28 + 14 26 + 24 24	0,5936624	104525
3d Exp.	211,1 211,1 211,1 33,25 212,0	- 32,75 44,15 32,75 212 212 45,3	1//, 85 167,0 178,35 178,75 166,7	20 + 33 19 + 81,5 20 + 93 20 + 172,5 18 + 193	0,3751069 0,3751069 0,3954320 0,4018165 0,3647221	093491 707357 698233 707918 689014	26 + 15 28 + 92 28 + 89,5 26 + 15,0	0,5582225 0,5582225 0,5925164 0,6003184 0,5514343	104101 105280 104636 105770 104187
General 1 Differenc	Mean of e of E2	f Propo spansio	n betw	Expansion, een Iron and	Brass,	0,00006963 <i>535</i>	= 0,000003545	-0, .495	00001050903

Results of Pyrometric Experiments.

TABLE III.

NOTE ON THE PRECEDING ARTICLE,

By Dr. Patterson.

An excellent work on Natural Philosophy, by Biot, received in this country since the communication of the preceding paper, contains an account of a series of experiments, made on the same subject, by Lavoisier and Laplace, in 1782, and which have not before been given to the public. According to these experiments, the expansion of iron, from the freezing to the boiling point, is 0,00122045 of its length, and that of brass 0,00188971. From these data, we readily calculate the following comparative statement.

Proportional Expansion for 1° Fahrenheit.

	For Iron.	For Brass.
According to Lavoisier and Laplace,	0,00000678.	0,000010498.
According to Mr. Hassler,	0,000 006 96.	0,000010498.
Difference,	0,00000018.	0,00 000 0011.

The correspondence of these results, obtained independently of each other, and by methods entirely different, must be considered as very satisfactory.



